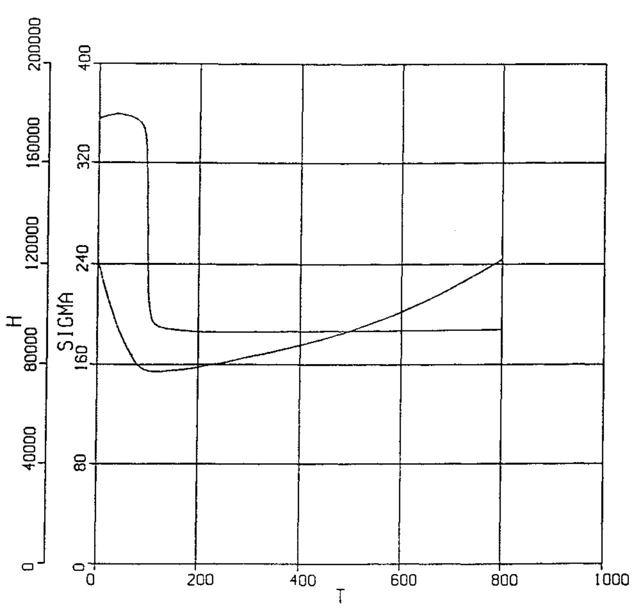

Fig. 1 Altitude h and bank angle σ vs time (full atmospheric flight).

energy, the controllability function is defined. By a state transformation, the considered system can be brought into a balanced form. In this form, the controllability function displays singular-value functions. If a singular-value function is small, then the corresponding state component is weakly controllable.

V. Numerical Results

The data used in the numerical experiments presented here are

$$g_1 = 10^{-4},\ g_2 = 10^{-5},\ \text{and}\ k_1 = k_2 = 1$$

Full atmospheric flight trajectory. Initial conditions (entry into the atmosphere) are $h_e = 121.896$ km, $\delta_e = -4.487$ deg, $\tau_e = -134.519$ deg, $\chi_e = 29.422$ deg, $V_e = 9894.121$ m/s, and $\gamma_e = -4.674$ deg. Desired final conditions (exit from the atmosphere) are $h_{de} = 122.139$ km and $V_{de} = 7615.411$ m/s.

Using the above data, simulations were carried out. To achieve the final conditions, the POWELL algorithm gives $\lambda_1^*(t_0) = 8.519 \times 10^{-8}$, $\lambda_2^*(t_0) = 0.999$, $\lambda_3^*(t_0) = -9.257 \times 10^{-3}$, $\lambda_4^*(t_0) = -0.1695$, $\lambda_5^*(t_0) = -2.658 \times 10^{-5}$, and $\lambda_6^*(t_0) = 0.887$. The final conditions are met, and this is shown by $\mathcal{E}(t_f) = 3.925 \times 10^{-3}$. The total flight duration is 800 s. We obtain the exit velocity $V_f = 7469.840$ m/s at the altitude $h_f = 122.126$ km and the wedge angle $\eta = 1.180$ deg. Figure 1 shows the time history of altitude and control. The spacecraft enters the atmosphere at altitude $h_e = 121.896$ km and exits at $h_f = 122.126$ km. The minimum altitude reached is 76.678 km. When the vehicle enters the atmosphere, the lift is maximally upward. In this phase, the bank angle is close to 360 deg, whereas in the atmospheric exit phase, the bank angle is close to 180 deg. These simulation results are similar to those obtained by Miele et al.[3,7]

Reduced atmospheric flight trajectory. Initial conditions are $h_e = 100.195$ km, $\delta_e = -3.155$ deg, $\tau_e = -132.161$ deg, $\chi_e = 29.587$ deg, $V_e = 9912.223$ m/s, and $\gamma_e = -3.428$ deg. Desired final conditions are $h_{de} = 100.285$ km and $V_{de} = 7509.270$ m/s.

In this case, the components of initial adjoint vector given by the POWELL algorithm are $\lambda_1^*(t_0) = 2.799 \times 10^{-7}$, $\lambda_2^*(t_0) = 2.278$, $\lambda_3^*(t_0) = -8.641 \times 10^{-3}$, $\lambda_4^*(t_0) = -5.447 \times 10^{-2}$, $\lambda_5^*(t_0) = -2.655 \times 10^{-5}$, and $\lambda_6^*(t_0) = 0.881$. By applying these values, the final conditions are reached, and $\mathcal{E}(t_f) = 1.930 \times 10^{-3}$ is obtained.

Using online computation, once a reference trajectory is determined by the optimization, it is possible to solve the trajectory tracking problem.[8,9]

VI. Conclusions

The optimal-control for the atmospheric flight phase of a spacecraft is treated. Due to the sensitivity of the optimization routine to the initialization of the adjoint vector, one has to choose the latter close to the optimum value. This is not compatible with the goods of the user.

By analyzing the vehicle's controllability, it is shown that the acceptable domain of initialization for the adjoint vector ensuring the convergence of the solution increases when the flight is considered in the lower altitude, i.e., when the controllability increases. The controllability analysis provides the domain where the optimization makes sense. Therefore, the optimal control should be applied to determine the reference trajectory only in the lower layers of the atmosphere. In the higher layers, the trajectory is not responsive to the input anyway.

Acknowledgment

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Shaping Time Response by State Feedback in Minimum-Phase Systems

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Introduction

IT is well known that for linear time-invariant multiple input–output state variable systems, controllability is a necessary and sufficient condition for eigenvalue assignment (pole placement) by means of state feedback.[1–3] For multivariable systems, the feedback gains are nonunique. This nonuniqueness has been utilized for shaping the response of the system by assigning selected eigenvectors.[4,5] On the other hand, for single-input systems, once the location of the closed-loop poles is specified, the feedback gains are unique. There is no additional freedom for shaping the response of the system. In this Note, we propose a systematic approach for shaping the time response of single-input, single-output systems.

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For single-input systems, typically, the dominant closed-loop poles are selected to meet the design specifications and the remaining poles are assigned to locations close to the origin for discrete-time systems or far in the left-half plane for the continuous-time systems. Clearly the above procedure for control design is heuristic. Further, since the above method does not take into account the effect of open-loop zeros on system response, it remains a trial-and-error method.

Often the primary design objective is to make the system follow a certain desired time response. In this Note it is shown that instead of specifying the closed-loop pole locations, one could specify the desired closed-loop time response and, using the technique proposed in the sequel, determine the closed-loop characteristic polynomial (equivalently the locations of the closed poles) that will achieve the desired response. The results in the sequel are developed for shaping the time response of single-input, single-output, discrete-time systems using state feedback. Further, it is assumed that the open-loop system has all its zeros inside the unit circle, i.e., the open-loop system has minimum phase.

Problem Formulation

From a unit pulse response data (possibly noisy), it is always possible to determine the parameters of the transfer function that best represent the given unit pulse response.[6] Using parameter optimization, up to $2n$ coefficients of the required nth-order transfer function are computed. Clearly, these techniques require complete freedom in selection of all coefficients of the numerator as well as denominator polynomials.[7,8] However, for the problem at hand, since we are trying to force the system to have a desired response by modifying it using (state) feedback, we cannot alter the locations of the open-loop plant zeros. Equivalently, the coefficients of the numerator polynomial remain unchanged after applying the feedback. Therefore, for the response-shaping problem, only n parameters (coefficients of the denominator polynomial) can be changed, making the problem more difficult than the conventional parameter estimation problem.

In the rest of this Note, we will use the following notation: X_{op} denotes an open-loop element, X_{cl} denotes a closed-loop element and X_d denotes a desired element. Assume that the given open-loop single-input, single-output plant is described by a strictly proper, minimum-phase, nth-order, z-domain transfer function:

$$H_{\text{op}}(z) = \frac{A_{\text{op}}(z)}{B_{\text{op}}(z)} = \frac{a_{\text{op}}(0) + a_{\text{op}}(1)z^{-1} + \cdots + a_{\text{op}}(n-1)z^{-(n-1)}}{1 + b_{\text{op}}(1)z^{-1} + \cdots + b_{\text{op}}(n)z^{-n}}$$

where the coefficient of the z^0 term in the denominator has been assumed to be unity without any loss of generality.

Further, assume that the design specifications are met by a convergent unit pulse response (to be provided by the designer) $\sum_{i=0}^{\infty} h_d(i)z^{-i}$. Ideally, one would like to know the entire unit pulse response, but assume that only the first ℓ significant (ℓ sufficiently large) samples of the desired pulse response

$$\boldsymbol{h}_d = [h_d(0), h_d(1), \ldots, h_d(\ell-1)]^T \tag{1}$$

are known.

Using parameter optimization techniques,[6,7] we can find a transfer function $H_d(z) = A_d(z)/B_d(z)$ that closely matches the response in Eq. (1). Knowing $H_d(z)$, a naive approach to closed-loop design would be to use state feedback for relocating the eigenvalues at the locations that corresponds to the roots of $B_d(z)$. However, in implementing $H_d(z)$, the numerator would also need to be modified. Since this is not feasible using feedback, the response of the compensated system may not be anywhere close to the desired response. Instead, we propose the following approach.

Assume that $H_d(z)$ is known in closed form (note that this assumption is made for clarity of presentation and is not a requirement for the proposed technique). Then knowing $H_d(z)$ and $A_{\text{op}}(z)$, determine $B_{\text{cl}}(z)$ such that $A_{\text{op}}(z)/B_{\text{cl}}(z)$ has the same response as the desired pulse response $\boldsymbol{h}_d$ in Eq. (1). Since it has been assumed that $H_d(z)$ is known in the closed form, the denominator polynomial $B_{\text{cl}}(z)$ can be written as

$$\frac{1}{B_{\text{cl}}(z)} = \frac{H_d(z)}{A_{\text{op}}(z)} = \frac{A_d(z)}{A_{\text{op}}(z)B_d(z)} \tag{2}$$

Equation (2) effectively states that the impulse response of $1/B_{\text{cl}}(z)$ closely matches that of $H_d(z)/A_{\text{op}}(z)$. Therefore, the problem of determining $B_{\text{cl}}(z)$ reduces to finding a denominator-only model (also known as all pole or autoregressive model) that matches the unit pulse response of the transfer function $A_d(z)/[A_{\text{op}}(z)B_d(z)]$ in Eq. (2) in the least-mean-square sense. The problem of estimating optimal parameters of an all-pole model from its impulse response was addressed in Ref. 8.

Although it has not been proven that the Steiglitz–McBride method in Ref. 8 will always give stable approximations for an arbitrarily specified pulse response data, extensive numerical simulations indicate that for convergent sequences it does indeed give stable rational approximations. In Eq. (2), due to the assumption that the open-loop system has minimum phase and $H_d(z)$ corresponds to a convergent sequence, the rational function $A_d(z)/[A_{\text{op}}(z)B_d(z)]$ will be stable and hence its pulse response will be a convergent sequence. Therefore, it is possible to determine a stable $B_{\text{cl}}(z)$ that approximates the response of $H_d(z)/A_{\text{op}}(z)$. Once $B_{\text{cl}}(z)$ is known, state feedback can be used to assign the closed-loop eigenvalues at the locations of the roots of B_{cl}, giving the closed-loop transfer function $H_{\text{cl}}(z) = A_{\text{op}}(z)/B_{\text{cl}}(z)$ with the desired unit pulse response.

Main Results

The next few paragraphs briefly outline the proposed algorithm (based on the Steiglitz–McBride algorithm) for the computation of the required all-pole model. We begin by defining

$$\hat{H}(z) = \frac{A_d(z)}{A_{\text{op}}(z)B_d(z)} = \hat{h}(0) + \hat{h}(1)z^{-1} + \cdots + \hat{h}(\ell-1)z^{-(\ell-1)} = \frac{1}{B_{\text{cl}}(z)}, \qquad \ell \gg n \tag{3}$$

where ℓ is approximately 10 times the order (n) of the system, and

$$\hat{\boldsymbol{h}} = [\hat{h}(0) \quad \hat{h}(1) \quad \cdots \quad \hat{h}(\ell-1)]^T \tag{4}$$

The pulse response $\hat{\boldsymbol{h}}$ will be the desired response for this step of the procedure. Let

$$\boldsymbol{h}^{(i)} = [h^{(i)}(0) \quad h^{(i)}(1) \quad \cdots \quad h^{(i)}(\ell-1)]^T$$

be the unit pulse response of $1/B_{\text{cl}}^{(i)}(z)$, where $1/B_{\text{cl}}^{(i)}(z)$ is the all-pole model obtained in the ith iteration.

Note that, by definition,

$$\hat{h}(n) = \hat{h}(n) * \delta(n)$$

where $*$ denotes the convolution operator and δ denotes the Dirac delta function. Also, $h^{(i)}(n) * b_{\text{cl}}^{(i)}(n) = \delta(n)$. Therefore, using the properties of the convolution operator,

$$\hat{h}(n) = h^{(i)}(n) * \hat{h}(n) * b_{\text{cl}}^{(i)}(n) \tag{5}$$

The convolution in Eq. (5) can be rewritten in matrix product form as given next. Define

$$H^{(i)} = \begin{bmatrix} h^{(i)}(0) & 0 & \cdots & 0 \\ h^{(i)}(1) & h^{(i)}(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h^{(i)}(\ell-1) & h^{(i)}(\ell-2) & \cdots & h^{(i)}(0) \end{bmatrix}$$

$$\hat{H} = \begin{bmatrix} \hat{h}(0) & 0 & \cdots & 0 \\ \hat{h}(1) & \hat{h}(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}(n) & \hat{h}(n-1) & \cdots & \hat{h}(0) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{h}(\ell-1) & \hat{h}(\ell-2) & \cdots & \hat{h}(\ell-n-1) \end{bmatrix} \tag{6}$$

where $H^{(i)} \in \mathbb{R}^{\ell\times\ell}$ and $H \in \mathbb{R}^{\ell\times(n+1)}$. Combining the definitions of H and $\hat{H}$ from Eq. (6) with the convolution in Eq. (5), we get

$$\hat{h} = H^{(i)}\hat{H}b_{\text{cl}} = [x^{(i)} \vdots X^{(i)}]\begin{bmatrix} 1 \\ b_{\text{cl}} \end{bmatrix} \tag{7}$$

where $\hat{x}^{(i)} \in \mathbb{R}^{\ell}$ denotes the first column and $\hat{X}^{(i)} \in \mathbb{R}^{\ell\times n}$ denotes the remaining columns of the product $H^{(i)}\hat{H}$ in Eq. (7); $b_{\text{cl}} = [b_{\text{cl}}(1), \ldots, b_{\text{cl}}(n)]^T$ and n is the order of the denominator polynomial.

Then, the following relation gives us the required iterative procedure for computing the coefficients of the closed-loop denominator polynomial $B_{\text{cl}}(z)$:

$$b_{\text{cl}}^{(i+1)} = [X^{(i)}]^{\dagger}[h^{(i)} - x^{(i)}] \tag{8}$$

where $[X^{(i)}]^{\dagger}$ is the pseudoinverse of $X^{(i)}$.

Clearly, $h^{(0)}$ (unknown) is required to start the iterations. In turn, $h^{(0)}$ is the pulse response of $1/B_{\text{cl}}(z)$, which, from Eqs. (3) and (4), is ideally equal to $\hat{h}$. Therefore, using Eqs. (3) and (4) and the properties of convolution,

$$\begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} = \hat{H}\begin{bmatrix} 1 \\ b_{\text{cl}}^{(0)} \end{bmatrix} = [\hat{h}_1 \vdots \hat{H}_2]\begin{bmatrix} 1 \\ b_{\text{cl}}^{(0)} \end{bmatrix} \tag{9}$$

where $\mathbf{0}$ is a zero vector of length $(\ell - 1)$ and $\hat{h}_1$ and $\hat{H}_2$ are respectively the first and remaining columns of $\hat{H}$. Rearranging the terms in Eq. (9), we get the initial estimate of the coefficients of $B_{\text{cl}}(z)$ as

$$b_{\text{cl}}^{(0)} = [\hat{H}_2]^{\dagger}\left(\begin{bmatrix} 1 \\ \mathbf{0} \end{bmatrix} - \hat{h}_1\right)$$

Knowing the initial estimate, the process is well started. For the next iteration, $H^{(1)}$ is constructed from the pulse response of $1/B_{\text{cl}}^{(0)}$.

Extensive numerical simulations suggest that the solution converges in five to six iterations. Knowing b_{cl} in Eq. (8) and therefore $B_{\text{cl}}(z)$, the necessary compensated system has the transfer function $A_{\text{op}}(z)/B_{\text{cl}}(z)$. The denominator polynomial $B_{\text{op}}(z)$ of the open system can now be modified to $B_{\text{cl}}(z)$ using state feedback.

It should be noted here that if the open-loop system has non-minimum phase, then the transfer function $H_d(z)/A_{\text{op}}(z)$ in Eq. (2) will become unstable. This will result in a divergent $\hat{h}$ in Eq. (4), leading to the estimation of an unstable $B_{\text{cl}}(z)$. Clearly, the approach presented in this Note cannot be applied to such systems.

Simulation Results

Example 1. The system for this example is a helicopter model.[9] The parameters of the system are given by

$$A = \begin{bmatrix} -9.1970 & 0.0 & -36.5400 & 7.3110 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -2.2940 & -821.9 & -18.7500 & 3.3170 \\ 0.7550 & -102.3 & 2.8680 & -0.6280 \end{bmatrix}$$

$$b = \begin{bmatrix} 589.00 \\ 0.00 \\ 517.50 \\ -79.14 \end{bmatrix}$$

$$c = [-10.0015 \quad 8.0650 \quad 1.5115 \quad -0.0849]$$

The system is discretized with a 0.1-s sampling period using zero-order hold. The transfer function of the corresponding discretized system is given by

$$H_{\text{op}}(z) = c(zI - A)^{-1}b = \frac{1 - 1.6614z^{-1} + 1.0076z^{-2} - 0.2733z^{-3}}{1 - 0.5871z^{-1} - 0.4229z^{-2} - 0.0222z^{-3} + 0.0574z^{-4}}$$

The discrete system has minimum phase, hence the proposed techniques are applicable to this system. For the purpose of illustration, suppose it is desired that the impulse response of the closed-loop system track that of the first-order system

$$H_d(z) = \frac{1}{1 - 0.85z^{-1}}$$

Since the system is of order 4, ℓ was chosen to be 50 (approximately 10 times ℓ). On applying the results presented here, the required closed-loop transfer function was found to be

$$H_{\text{cl}}(z) = \frac{1 - 1.6614z^{-1} + 1.0076z^{-2} - 0.2733z^{-3}}{1 - 2.4614z^{-1} + 2.3367z^{-2} - 1.0794z^{-3} + 0.2186z^{-4}}$$

Note that the numerator polynomials of open-loop and closed-loop systems are the same. The desired first-order response and the response of the closed-loop system as well as the error between the two are shown in Fig. 1. The remarkably low maximum error is of the order 10^{-14}.

Example 2. The system for this example is the same helicopter model as in Example 1. For the purpose of illustration, it is

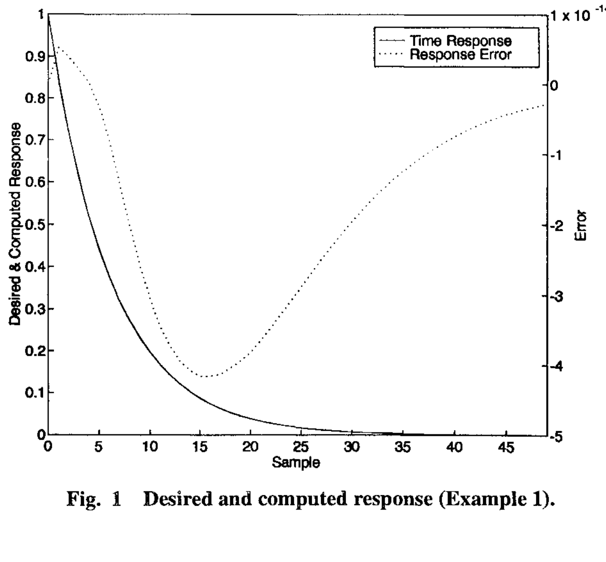

Fig. 1 Desired and computed response (Example 1).

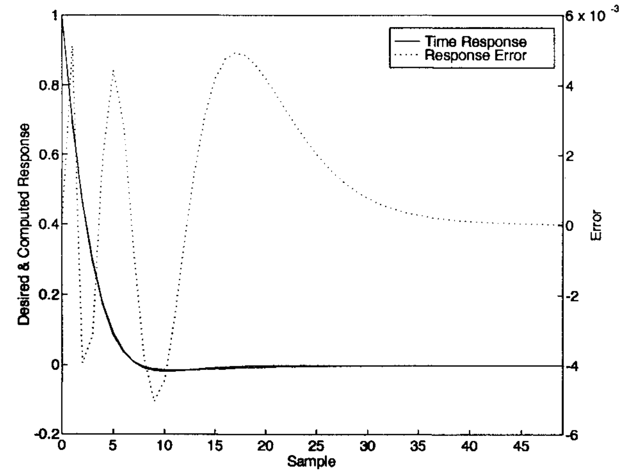

Fig. 2 Desired and computed response (Example 2).

desired that the impulse response of the closed-loop system track that of

$$H_d(z) = \frac{1 - 0.7z^{-1}}{1 - 1.4z^{-1} + 0.5125z^{-2}}$$

which corresponds to a discretized second-order continuous system with damping ratio 0.8455 and natural frequency 3.9531 rad.

The required closed-loop transfer function was found to be

$$H_{cl}(z) = \frac{1 - 1.6614z^{-1} + 1.0076z^{-2} - 0.2733z^{-3}}{1 - 2.3562z^{-1} + 2.1734z^{-2} - 0.9716z^{-3} + 0.1822z^{-4}}$$

The desired second-order impulse response and the response of the closed-loop system along with the error are shown in Fig. 2. For the sake of brevity, only the final results are shown. Detailed results and software used for the simulations can be obtained from the authors.

Concluding Remarks

In this Note, we considered the problem of time response shaping in single-input, single-output systems by means of full state feedback. Under a minimum-phase assumption on the open-loop plant, it was shown that it is possible to determine the closed-loop characteristic polynomial such that the impulse response of the compensated system closely matches a prescribed impulse response. The case of non-minimum-phase systems as well as multivariable systems requires a different treatment and will be reported elsewhere.

Acknowledgment

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Experimental Control of a Single-Link Flexible Arm Incorporating Electrorheological Fluids

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Introduction

RECENTLY, the emergence of several vibration control technologies exploiting smart materials has been reported in the literature.[1] Smart materials with adaptive mechanical properties have the potential to stimulate the evolution of a new generation of flexible manipulators with enhanced performance characteristics. This technical brief focuses upon one aspect of this embryonic field by presenting a proof-of-concept investigation on the vibration control of a single-link flexible manipulator incorporating embedded electrorheological (ER) fluids.[2,3] Since the phenomenologic behavior associated with the interaction of the structure and the embedded ER fluid is complex, simplifying assumptions have been incorporated in the modeling process to furnish equations of motion that are analogous in mathematical structure to those encountered in beams fabricated from homogeneous monolithic materials. The phenomenologic equation of motion, based on these simplifying assumptions and experimental observations, permits a distributed ER fluid actuator to be implemented. The control force associated with the ER fluid actuator accounts for the change in the global damping and stiffness properties of the structure when an electric field is applied to the ER fluid domains. The ER fluid actuator is then incorporated with the typical output feedback controller associated with two collocated angular position and velocity sensors to evaluate the control performances of the manipulator. The step responses such as settling time, overshoot, and tip deflection are evaluated and compared for the condition of zero and nonzero electric field states. Furthermore, an attempt to increase the robustness of the conventional proportional-plus-derivative output feedback control system is made herein by imposing a special class of external disturbances, which have excitation frequencies very close to the natural frequencies of the flexible arm.

Controller Design

Consider the elastodynamic flexural response in the horizontal plane of a single-link flexible arm featuring an embedded ER fluid as shown in Fig. 1. The uniform beam is attached to the rotating hub. Previous publications have shown[4,5] that the vibrational characteristics of the flexible beam incorporating ER fluids, such as natural frequencies and damping capacities, can be tuned by imposing electric fields of the fluid domain. It has also been shown[6] that the phases of the mode shapes of ER embedded flexible beams are all near 0 deg and 180 deg at both 0 and 3 kV/mm, and the mode shapes are nearly identical for a certain range of electric fields.

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